Embedding reaction systems into link-calculus

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joint work with

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Roadmap

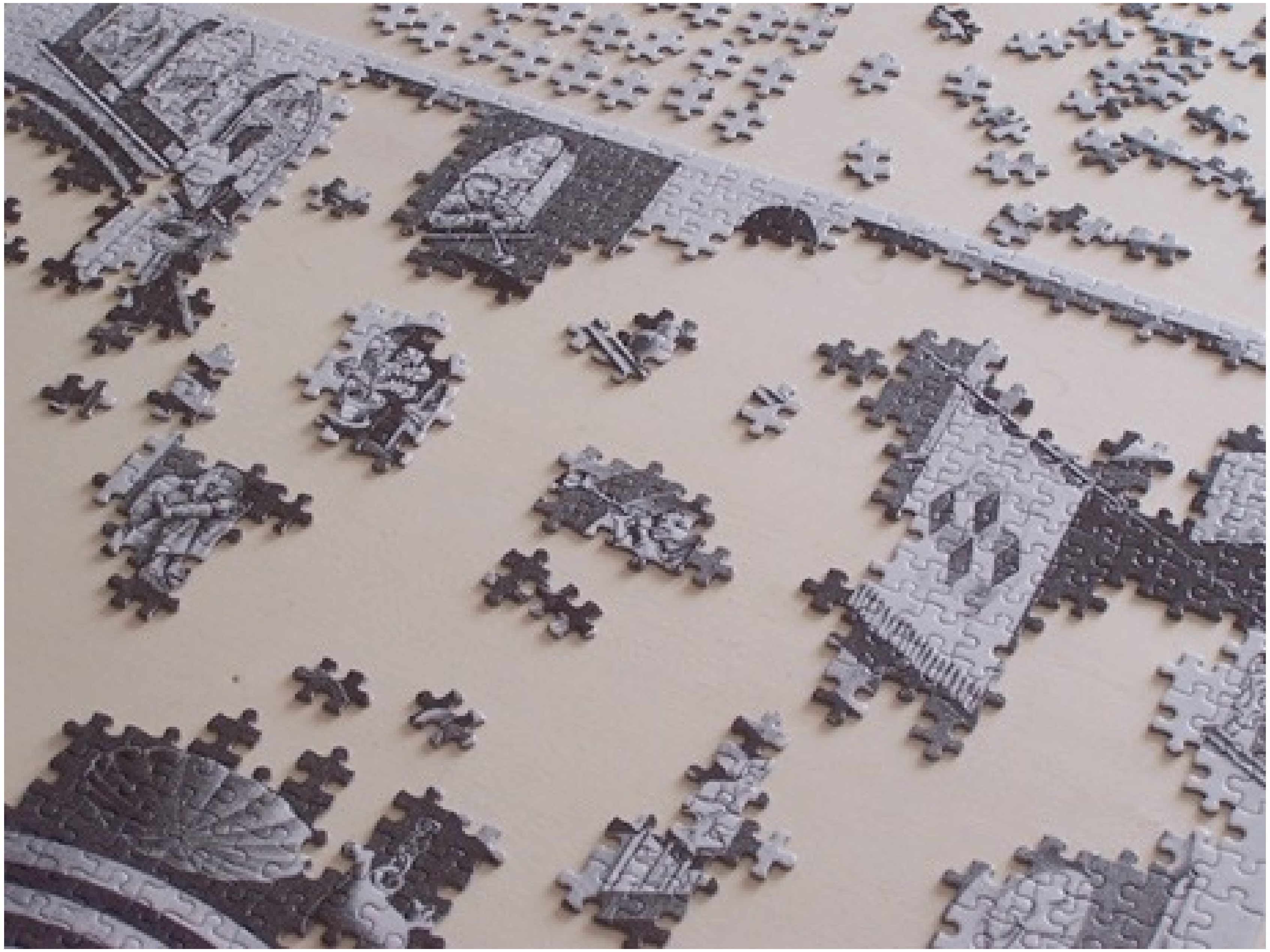
- ✓ An open multi-party calculus
  (joint work with Roberto Bruni and Chiara Bodei -Univ. of Pisa-)
- ✓ Encoding reactions
- ✓ Encoding entities
- ✓ Encoding contexts
- ✓ Enhancing expressivity for reaction systems
- ✓ Conclusion and future work

Interaction

An interaction is an action
by which
(communicating) processes
can influence each other.

Multiparty interaction

An interaction is multiparty when
it involves two or more processes



Open interaction

An interaction is open when
the number of involved processes is not fixed

Notation

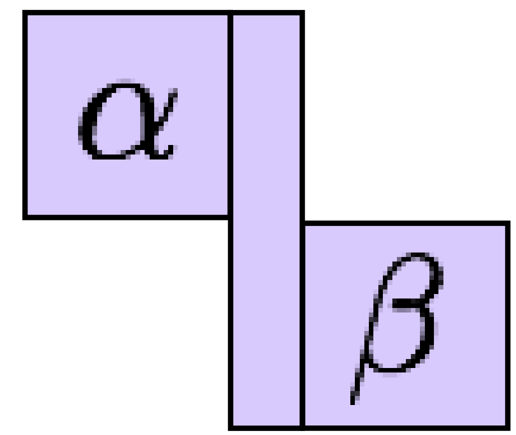
a interaction over a

τ silent interaction

$\Box$ free “slot”, accepting
any interaction (only in labels)

Link

$^{\alpha}\backslash_{\beta}$ From α to β

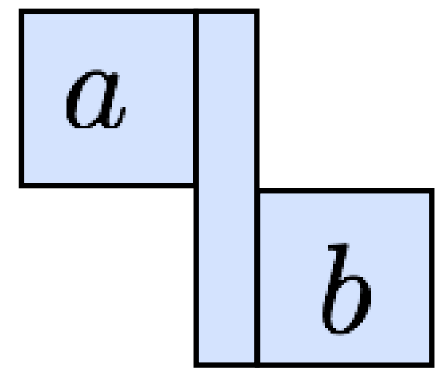


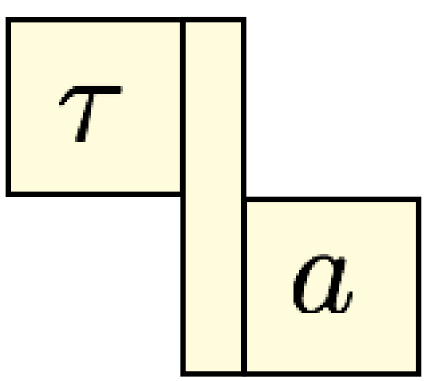
Valid:

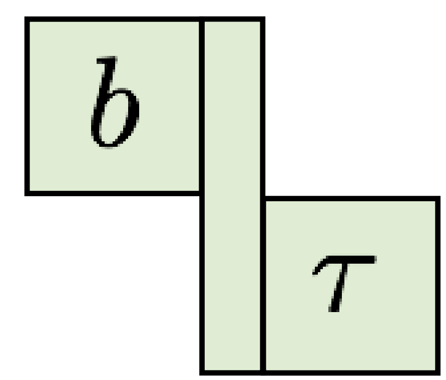
if it is **virtual** $^{\square}\backslash_{\square}$

if it is **solid** $\alpha, \beta \neq \square$

Example: three party







Link chain

$$^{\alpha_1}\backslash_{\beta_1} \ {}^{\alpha_2}\backslash_{\beta_2} \ \cdots \ {}^{\alpha_n}\backslash_{\beta_n}$$

$\mathcal{C}$ is the set of channel names

such that:

$$\beta_i, \alpha_{i+1} \in \mathcal{C} \quad \text{implies } \beta_i = \alpha_{i+1}$$

$$\beta_i = \tau \qquad \text{iff } \alpha_{i+1} = \tau$$

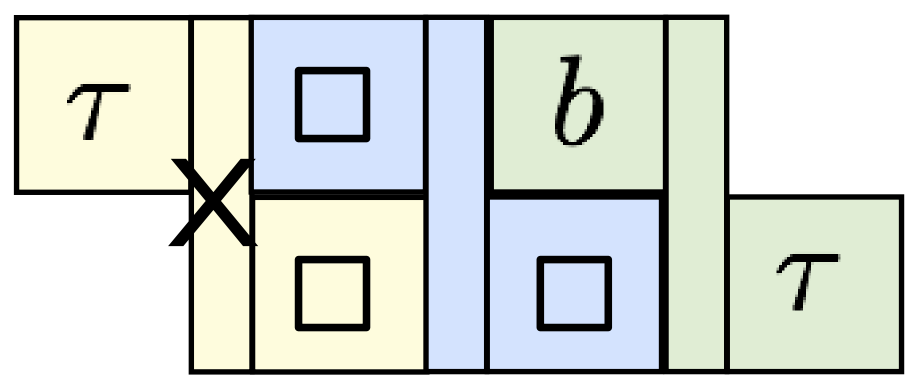
Link chain: terminology

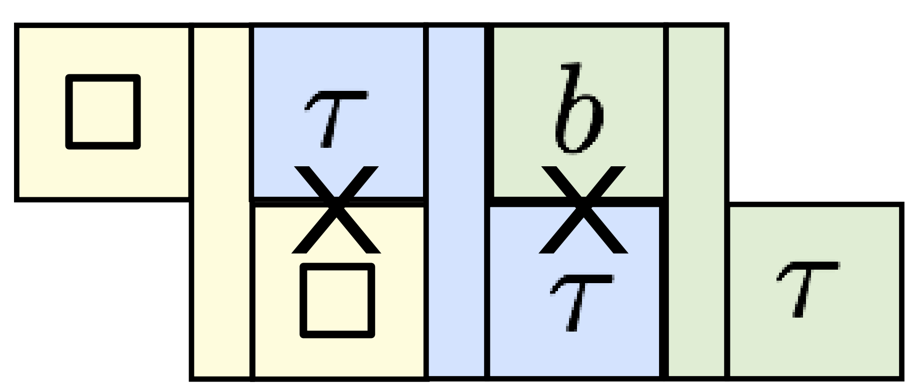
$$^{\alpha_1}\backslash_{\beta_1}\ ^{\alpha_2}\backslash_{\beta_2}\ \ldots\ ^{\alpha_n}\backslash_{\beta_n}$$

Solid:

if all its links are so

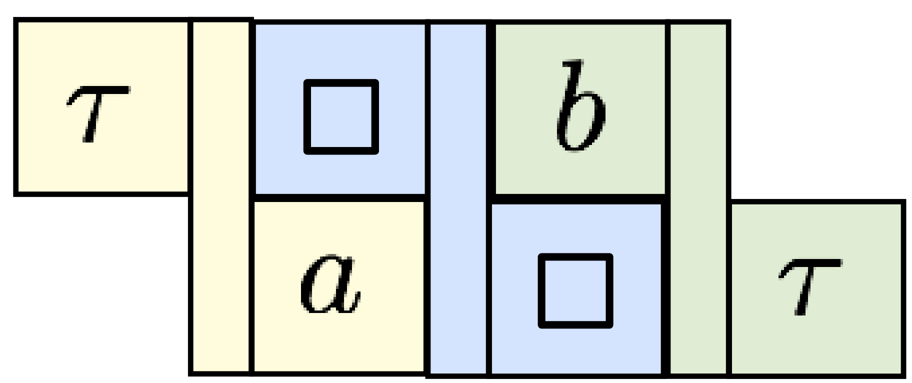
Counter-examples



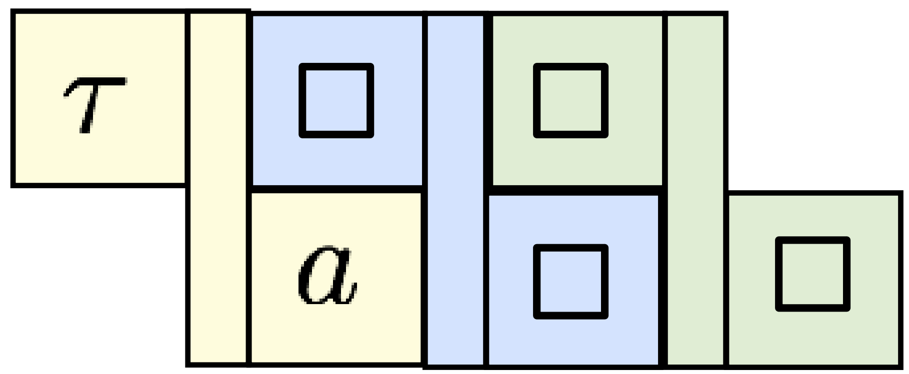


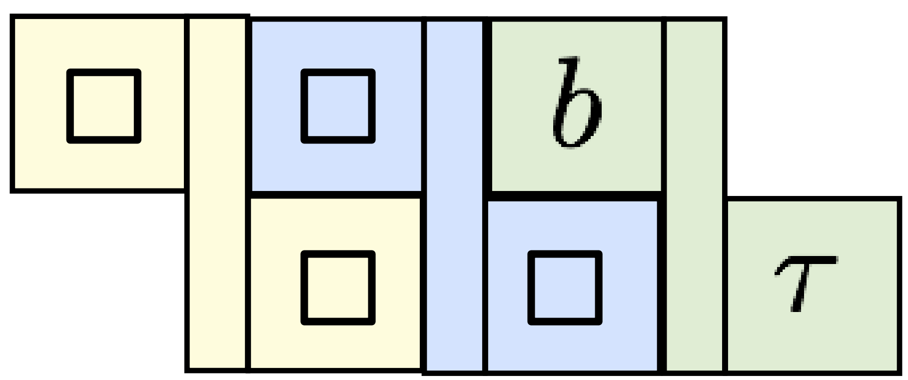
Examples: non solid

Virtual links
can be read as missing pieces of the puzzle $\square \backslash \square$



Examples: merge





Merge

$$s = \ell_1 ... \ell_n \qquad\qquad s' = \ell'_1 ... \ell'_n$$

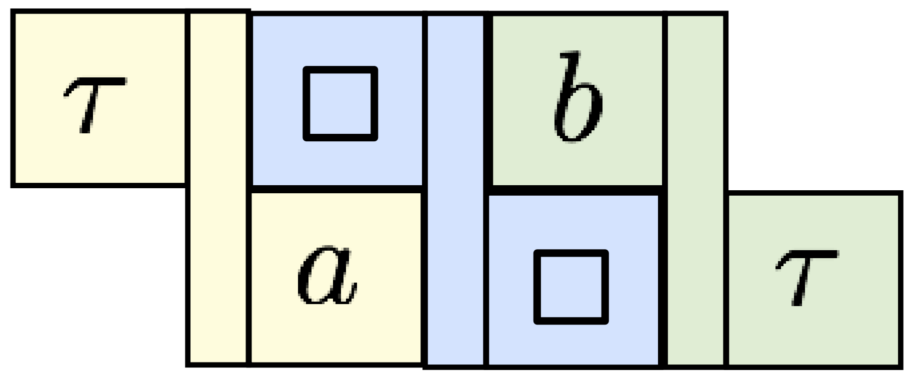
$$s \bullet s' \quad \triangleq \quad (\ell_1 \bullet \ell'_1) \cdots (\ell_n \bullet \ell'_n)$$

$${}^{\alpha}\backslash_{\beta} \bullet {}^{\alpha'}\backslash_{\beta'} \quad \triangleq \quad {}^{(\alpha \bullet \alpha')}\backslash_{(\beta \bullet \beta')}$$

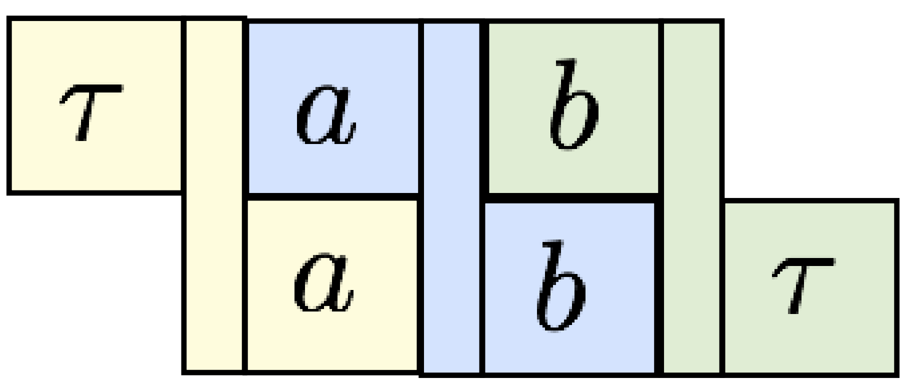
$$\alpha \bullet \beta \quad \triangleq \quad \begin{cases} \alpha & \text{if } \beta = \square \\ \beta & \text{if } \alpha = \square \end{cases}$$

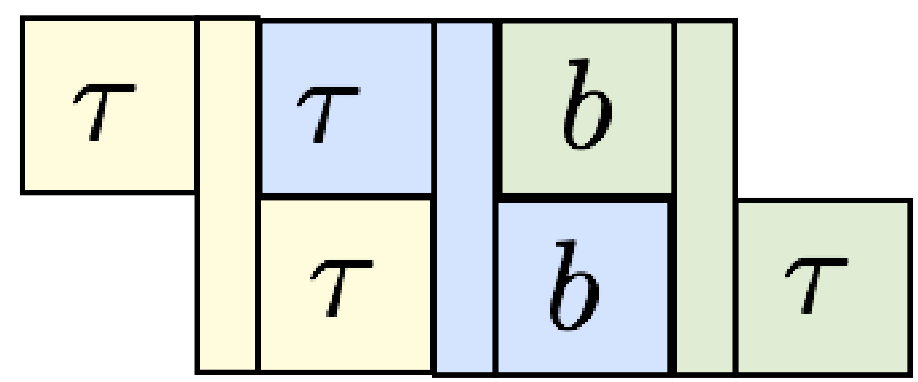
The result is undefined if the outcome is not valid

Examples: restriction

(νa) 

$= \perp$

(νa) 

$=$ 

Restriction

$$(\nu\, a)s \;\triangleq\; \begin{cases} ((\nu\, a)\ell_1)\ldots((\nu\, a)\ell_n) & \text{if } a \text{ is } \textit{matched} \text{ in } s \\ \perp & \text{otherwise} \end{cases}$$

$$(\nu\, a)^{\alpha}\backslash_{\beta} \;\triangleq\; {}^{((\nu\, a)\alpha)}\backslash_{((\nu\, a)\beta)}$$

$$(\nu\, a)\alpha \;\triangleq\; \begin{cases} \tau & \text{if } \alpha = a \\ \alpha & \text{otherwise} \end{cases}$$

Equivalence relation over link chains

(the black tie)

$$s^{\square}\backslash_{\square} \;\blacktriangleright\!\!\blacktriangleleft\; s \qquad\qquad s_1{}^{\square}\backslash^{\square}_{\square}\backslash_{\square} s_2 \;\blacktriangleright\!\!\blacktriangleleft\; s_1{}^{\square}\backslash_{\square} s_2$$

$${}^{\square}\backslash_{\square} s \;\blacktriangleright\!\!\blacktriangleleft\; s \qquad\qquad s_1{}^{\alpha}\backslash^{a}_{a}\backslash_{\beta} s_2 \;\blacktriangleright\!\!\blacktriangleleft\; s_1{}^{\alpha}\backslash^{\square}_{a}\backslash^{a}_{\square}\backslash_{\beta} s_2$$

link-calculus syntax

$$P, Q \ ::= \ \mathbf{0} \mid \ell.P \mid P + Q \mid P|Q \mid (\nu\, a)P \mid P[\phi] \mid A$$

null action

prefix
(link prefix)

choice

parallel

restriction

relabelling

recursion

very closed to the CCS syntax

(Relevant) SOS rules

the length of the link chains (of a transition) is decided by the semantics

$$\frac{s \bowtie \ell}{\ell.P \xrightarrow{s} P} \ (Act)$$

$$\frac{P \xrightarrow{s} P'}{(\nu\, a)P \xrightarrow{(\nu\, a)s} (\nu\, a)P'} \ (Res)$$

$$\frac{P \xrightarrow{s} P'}{P|Q \xrightarrow{s} P'|Q} \ (Lpar)$$

$$\frac{P \xrightarrow{s} P' \qquad Q \xrightarrow{s'} Q'}{P|Q \xrightarrow{s \bullet s'} P'|Q'} \ (Com)$$

Example

$$P \triangleq {}^{\tau}\backslash_{a}.P_1|(\nu\, b)Q\ ,\ Q \triangleq {}^{b}\backslash_{\tau}.P_2|{}^{a}\backslash_{b}.\mathbf{0}$$

$$\dfrac{\dfrac{}{{}^{\tau}\backslash_{a}.P_1 \xrightarrow{{}^{\tau}\backslash^{\square}_{a}\backslash^{\square}_{\square}\backslash_{\square}} P_1}\,(Act) \qquad \dfrac{\dfrac{\dfrac{}{{}^{b}\backslash_{\tau}.P_2 \xrightarrow{{}^{\square}\backslash^{\square}_{\square}\backslash^{b}_{\square}\backslash_{\tau}} P_2}\,(Act) \qquad \dfrac{}{{}^{a}\backslash_{b}.\mathbf{0} \xrightarrow{{}^{\square}\backslash^{a}_{\square}\backslash^{\square}_{b}\backslash_{\square}} \mathbf{0}}\,(Act)}{Q \xrightarrow{{}^{\square}\backslash^{a}_{\square}\backslash^{b}_{b}\backslash_{\tau}} P_2|\mathbf{0}}\,(Com)}{(\nu\, b)Q \xrightarrow{{}^{\square}\backslash^{a}_{\square}\backslash^{\tau}_{\tau}\backslash_{\tau}} (\nu\, b)(P_2|\mathbf{0})}\,(Res)}{P \xrightarrow{{}^{\tau}\backslash^{a}_{a}\backslash^{\tau}_{\tau}\backslash_{\tau}} P_1|(\nu\, b)(P_2|\mathbf{0})}\,(Com)$$

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http://linkcalculus.di.unipi.it

Roadmap

- ✓ An open multi-party calculus
- ✓ Encoding reaction systems
- ✓ Encoding entities
- ✓ Encoding contexts
- ✓ Enhancing expressivity for reaction systems
- ✓ Conclusion and future work

Reaction Systems

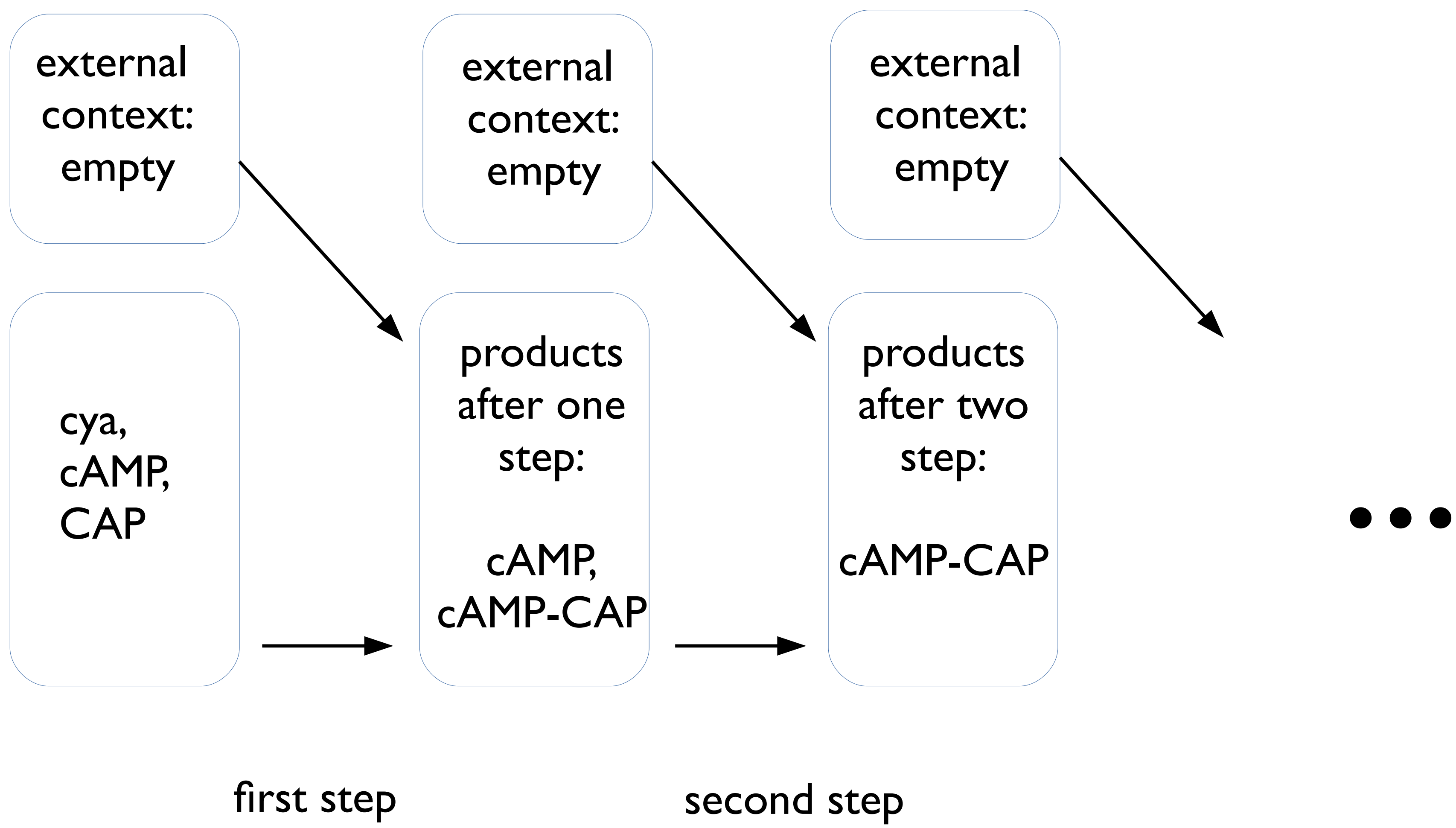
A reaction system is a set of rules of the type:

$$a = (R, I, P)$$

Reactants Inhibitors Products

$$(\{cAMP, CAP\}, \{glucose\}, \{cAMP\text{–}CAP\})$$

Reaction Systems



always are applied
(when possible)
all together

$$(\{cya\}, \{...\}, \{cAMP\})$$

$$(\{cAMP, CAP\}, \{glucose\}, \{cAMP\text{-}CAP\})$$

The *chained* link-calculus

Is a version of the link-calculus where prefixes are link chains.

syntax $P, Q ::= \sum_{i \in I} v_i.P_i \mid P|Q \mid (\nu a)P \mid P[\phi] \mid A$

link chain prefix $v = \ell_1 ... \ell_n$

relevant semantic rule

$$\frac{v \bowtie v_j}{\sum_{i \in I} v_i.P_i \xrightarrow{v} P_j} \; (Sum)$$

Encoding reaction systems:

usage of the names

entities assume different behaviours:

reaction systems		link-calculus
a	present acting as reagent or inhibitor	a
a	absent	$\overline{a}$
a	products	ã

Encoding reactions

(when applicable)

assuming a rs with only 2 reactions, and 5 entities:

reaction 1 $$(\{cya\}, \{...\}, \{cAMP\})$$

reaction 2 $$(\{cAMP, CAP\}, \{glucose\}, \{cAMP\text{-}CAP\})$$

encoding the two reactions as link-calculus processes

reaction 1 $$P_1 \triangleq \tau \backslash^{\square}_{cya_i} \backslash^{cya_o}_{\square} \backslash^{\square}_{\widetilde{cAMP}_i} \backslash^{\widetilde{cAMP}_o}_{\square} \backslash_{r_2}.P_1 \quad + \ \dots$$

reaction 2
$$P_2 \triangleq r_2 \backslash^{\square}_{cAMP_i} \backslash^{cAMP_o}_{\square} \backslash^{\square}_{CAP_i} \backslash^{CAP_o}_{\square} \backslash^{\square}_{\overline{glucose}_i} \backslash^{\overline{glucose}_o}_{\square} \backslash^{\square}_{\widetilde{cAMP\text{-}CAP}_i} \backslash^{\widetilde{cAMP\text{-}CAP}_o}_{\square} \backslash_{\tau}.P_2 \quad + \ \dots$$

Encoding reactions

when the reaction is not applicable, we still execute the process encoding the reaction

reaction 2 $(\{cAMP, CAP\}, \{glucose\}, \{cAMP\text{-}CAP\})$

$$P_2 \triangleq \ldots +$$

$$r_2 \backslash^{\square}_{glucose_i} \backslash^{glucose_o}_{\square} \backslash_{\tau} . P_2$$

$$+$$

$$r_2 \backslash^{\square}_{\overline{cAMP}_i} \backslash^{\overline{cAMP}_o}_{\square} \backslash_{\tau} . P_2$$

$$+$$

$$r_2 \backslash^{\square}_{\overline{CAP}_i} \backslash^{\overline{CAP}_o}_{\square} \backslash_{\tau} . P_2$$

Encoding reactions

the link chain prefixes of the two reactions can be linked

$$\tau\backslash^{\Box}_{cya_i}\backslash^{cya_o}_{\Box}\backslash^{\Box}_{\widetilde{cAMP}_i}\backslash^{\widetilde{cAMP}_o}_{\Box}\backslash r_2 \quad r_2\backslash^{\Box}_{cAMP_i}\backslash^{cAMP_o}_{\Box}\backslash^{\Box}_{CAP_i}\backslash^{CAP_o}_{\Box}\backslash^{\Box}_{\overline{glucose}_i}\backslash^{\overline{glucose}_o}_{\Box}\backslash^{\Box}_{\widetilde{cAMP\text{-}CAP}_i}\backslash^{\widetilde{cAMP\text{-}CAP}_o}_{\Box}\backslash\tau$$

Encoding reactions

the link chain prefixes of the two reactions can be linked

$$\tau\backslash^{\square}_{cya_i}\backslash^{cya_o}_{\square}\backslash^{\square}_{\widetilde{cAMP}_i}\backslash^{\widetilde{cAMP}_o}_{\square}\backslash_{r_2}\quad {}^{r_2}\backslash^{\square}_{cAMP_i}\backslash^{cAMP_o}_{\square}\backslash^{\square}_{CAP_i}\backslash^{CAP_o}_{\square}\backslash^{\square}_{\overline{glucose}_i}\backslash^{\overline{glucose}_o}_{\square}\backslash^{\square}_{\widetilde{cAMP\text{-}CAP}_i}\backslash^{\widetilde{cAMP\text{-}CAP}_o}_{\square}\backslash\tau$$

Encoding reactions

the link chain prefixes of the two reactions can be linked
(*forming a sort of communication backbone*):

$$\tau \backslash^{\square}_{cya_i} \backslash^{cya_o}_{\square} \backslash^{\square}_{\widetilde{cAMP}_i} \backslash^{\widetilde{cAMP}_o}_{\square} \backslash r_2 \quad r_2 \backslash^{\square}_{cAMP_i} \backslash^{cAMP_o}_{\square} \backslash^{\square}_{CAP_i} \backslash^{CAP_o}_{\square} \backslash^{\square}_{\overline{glucose}_i} \backslash^{\overline{glucose}_o}_{\square} \backslash^{\square}_{\widetilde{cAMP\text{-}CAP}_i} \backslash^{\widetilde{cAMP\text{-}CAP}_o}_{\square} \backslash \tau$$

what is still missing is the contribution of the single entities (molecules)

Encoding reactions

the link chain prefixes of the two reactions can be linked
(*forming a sort of communication backbone*):

$$\tau \backslash^{\square}_{cya_i} \backslash^{cya_o}_{\square} \backslash^{\square}_{\widetilde{cAMP}_i} \backslash^{\widetilde{cAMP}_o}_{\square} \backslash r_2 \qquad r_2 \backslash^{\square}_{cAMP_i} \backslash^{cAMP_o}_{\square} \backslash^{\square}_{CAP_i} \backslash^{CAP_o}_{\square} \backslash^{\square}_{\overline{glucose}_i} \backslash^{\overline{glucose}_o}_{\square} \backslash^{\square}_{\widetilde{cAMP\text{-}CAP}_i} \backslash^{\widetilde{cAMP\text{-}CAP}_o}_{\square} \backslash \tau$$

what is still missing is the contribution of the single entities (molecules)

Roadmap

- ✔ An open multi-party calculus
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Enconding entities

When the entity is present or produced

$$P_{cya} \quad \triangleq \quad \begin{array}{l} \sum_{h,k\geq 0} ({}^{cya_i}\backslash^{\Box}_{cya_o}\backslash\Box)^h ({}^{c\tilde{y}a_i}\backslash^{\Box}_{c\tilde{y}a_o}\backslash\Box)^k . P_{cya} \\ + \\ \sum_{h\geq 0} ({}^{cya_i}\backslash^{\Box}_{cya_o}\backslash\Box)^h . \overline{P_{cya}} \end{array}$$

Enconding entities

When the entity is absent or produced

$$
\begin{aligned}
\overline{P_{cya}} \quad \triangleq \quad & \textstyle\sum_{h,k\geq 0}\left(\overline{cya}_i \backslash \frac{\square}{\overline{cya}_o} \backslash \square\right)^h \left(c\tilde{y}a_i \backslash \frac{\square}{c\tilde{y}a_o} \backslash \square\right)^k . P_{cya} \\
& + \\
& \textstyle\sum_{h\geq 0}\left(\overline{cya}_i \backslash \frac{\square}{\overline{cya}_o} \backslash \square\right)^h . \overline{P_{cya}}
\end{aligned}
$$

The enconding:

reactions + entities

$$\tau\,{}^{\square}\backslash_{cya_i}\,{}^{cya_o}\backslash_{\square}\,{}^{\square}\backslash_{\widetilde{cAMP}_i}\,{}^{\widetilde{cAMP}_o}\backslash_{\square}\,\backslash_{r_2} \quad r_2\,{}^{\square}\backslash_{cAMP_i}\,{}^{cAMP_o}\backslash_{\square}\,{}^{\square}\backslash_{CAP_i}\,{}^{CAP_o}\backslash_{\square}\,{}^{\square}\backslash_{\overline{glucose}_i}\,{}^{\overline{glucose}_o}\backslash_{\square}\,{}^{\square}\backslash_{\widetilde{cAMP\text{-}CAP}_i}\,{}^{\widetilde{cAMP\text{-}CAP}_o}\backslash_{\square}\,\backslash_{\tau}$$

$$P_{CAP} \triangleq {}^{CAP_i}\backslash_{CAP_o}.P_{CAP}$$

$$P_{\overline{glucose}} \triangleq {}^{\overline{glucose}_i}\backslash_{\overline{glucose}_o}.P_{\overline{glucose}}$$

$$P_{cya} \triangleq {}^{cya_i}\backslash_{cya_o}.P_{cya}$$

$$P_{cAMP} \triangleq {}^{\widetilde{cAMP}_i}\backslash^{\square}_{\widetilde{cAMP}_o}\,{}^{cAMP_i}\backslash_{\square}\,\backslash_{cAMP_o}.P_{cAMP}$$

$$P_{cAMP\text{-}CAP} \triangleq {}^{\widetilde{cAMP\text{-}CAP}_i}\backslash_{\widetilde{cAMP\text{-}CAP}_o}.P_{cAMP\text{-}CAP}$$

encoding the entities

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Enconding reaction systems:

(more) usage of the names

entities assume different roles:

reaction systems		link-calculus
	...	
a	provided by the context	$\hat{\mathrm{a}}$
a	not provided by the context (absence)	$\underline{\mathrm{a}}$

Adding contexts

how a context behaves

$$Cxt^{n}_{cya} \triangleq \begin{cases} cxt_j \backslash^{\Box}_{c\hat{y}a_i} \backslash^{c\hat{y}a_o}_{\Box} \backslash cxt_{j+1} . Cxt^{n+1}_{cya} \\ \\ cxt_j \backslash^{\Box}_{\underline{cya}_i} \backslash^{\underline{cya}_o}_{\Box} \backslash cxt_{j+1} . Cxt^{n+1}_{cya} \end{cases}$$

$$Cxt_{cya} \triangleq Cxt^{1}_{cya}$$

Make contexts synchronise with entities

$$
\begin{aligned}
P_{cya} \quad \triangleq \quad & \sum_{h,k\geq 0}\left(cya_i \backslash^{\Box}_{cya_o} \backslash \Box\right)^h \; c\hat{y}a_i \backslash^{\Box}_{c\hat{y}a_o} \backslash \Box \left(c\tilde{y}a_i \backslash^{\Box}_{c\tilde{y}a_o} \backslash \Box\right)^k . P_{cya} \\
& + \\
& \sum_{h\geq 0,k\geq 1}\left(cya_i \backslash^{\Box}_{cya_o} \backslash \Box\right)^h \; \underline{cya}_i \backslash^{\Box}_{\underline{cya}_o} \backslash \Box \left(c\tilde{y}a_i \backslash^{\Box}_{c\tilde{y}a_o} \backslash \Box\right)^k . P_{cya} \\
& + \\
& \sum_{h\geq 0}\left(cya_i \backslash^{\Box}_{cya_o} \backslash \Box\right)^h \; \underline{cya}_i \backslash_{\underline{cya}_o} . \overline{P_{cya}}
\end{aligned}
$$

Make contexts synchronise with reactions

$$
\begin{aligned}
P_2 \;\triangleq\;& \ldots \;+ \\
& {r_2}\backslash^{\square}_{glucose_i}\backslash^{glucose_o}_{\square}\backslash_{\tau}.P_2 \\
+\; & {r_2}\backslash^{\square}_{\overline{cAMP}_i}\backslash^{\overline{cAMP}_o}_{\square}\backslash_{\tau}.P_2 \\
+\; & {r_2}\backslash^{\square}_{\overline{CAP}_i}\backslash^{\overline{CAP}_o}_{\square}\backslash_{cxt_1}.P_2
\end{aligned}
$$

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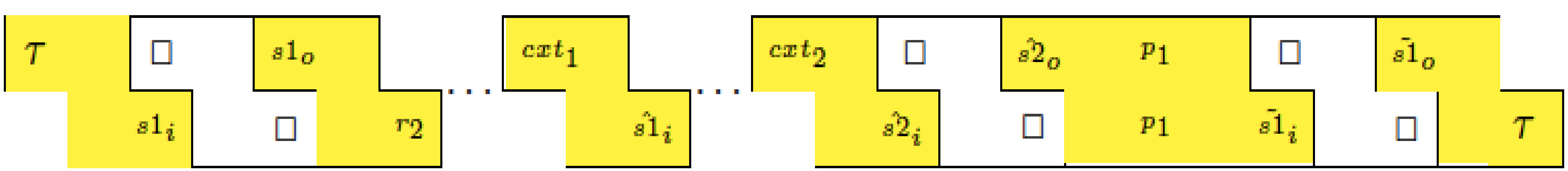
What we gain:

- ✔ recursive contexts
- ✔ modeling mutating entities
- ✔ communicating reaction systems: for example, the lac operon system (that depends on the presence or absence of the glucose) can be connected with the system producing the glucose.
- ✔ modeling style: backbone + resources: the processes encoding the reactions and the context form the backbone; processes encoding entities provide the resources.



Future work

We would like to:

- ✓ model two communicating reaction systems;
- ✓ model a reaction system with mutating entities;
- ✓ exploit the nature of process algebra to define properties of reaction systems;
- ✓ ...

THANKS FOR YOUR ATTENTION!

questions ?... any suggestion ?

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questions ?

... any suggestion ?